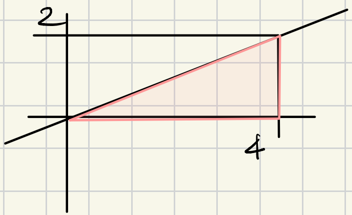
5.4 Changing the order of integration

We have already seen that it may be possible to integrate over a region in two ways: x first, y second; or y first, x second. Sometimes we can't do the integration one way round, but we can do it the other way.

Example: $\displaystyle\int_0^2 \int_{2y}^4 e^{x^2}\,dx\,dy$

Problem: we don't know how to do $\int e^{x^2}dx$



Do it the other way

$$\int_0^4 \int_0^{\frac{1}{2}x} e^{x^2}\,dy\,dx$$

$$= \int_0^4 \left[y e^{x^2} \right]_0^{\frac{1}{2}x} dx$$

$$= \int_0^4 \tfrac{1}{2}x e^{x^2}\,dx = \left[\frac{e^{x^2}}{4} \right]_0^4$$

$$= \frac{e^{16}-1}{4}$$

Mean value inequality and equality for integrals

If $m \le f(x,y) \le M$ for all (x,y) in D then

$$m\,A(D) \le \iint_D f\,dA \le M\,A(D)$$

where $A(D)$ is the area of D.

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$$\frac{1}{\sqrt{3}} \le \iint_{[0,1]\times[0,1]} \frac{1}{\sqrt{1+x^6+y^8}}\,dA \le 1$$

Theorem 5. If f is continuous then for some (x_0,y_0) in D we have

$$\iint_D f(x,y)\,dA = f(x_0,y_0)\,A(D)$$